

Katalin Marton

Abbas El Gamal

Stanford University

Withits 2010

Brief Bio

- Born in 1941, Budapest Hungary
- PhD from Eötvös Loránd University in 1965
- Department of Numerical Mathematics, Central Research Institute for Physics, Budapest, 1965–1973
- Mathematical Institute of the Hungarian Academy of Sciences, 1973–present
- Visited Institute of Information Transmission, Moscow, USSR, 1969
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- Research contributions and interests:
 - ▶ Information Theory
 - ▶ Measure concentration
 - ▶ Applications in Probability Theory



Selected Contributions to Information Theory

- Broadcast channels:

J. Körner, K. Marton, "Comparison of two noisy channels," *Colloquia Mathematica Societatis, János Bolyai*, 16, *Topics in Information Theory*, North Holland, pp. 411-424, 1977

J. Körner, K. Marton, "Images of a set via two different channels and their role in multiuser communication," *IEEE Transactions on Information Theory*, IT-23, pp. 751-761, Nov. 1977

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- **Rate distortion theory:**

K. Marton, "Asymptotic behavior of the rate distortion function of discrete stationary processes," *Problomy Peredachi Informatsii*, VII, 2, pp. 3-14, 1971

K. Marton, "On the rate distortion function of stationary sources," *Problems of Control and Information Theory*, 4, pp. 289-297, 1975

- **Error exponents:**

K. Marton, "Error exponent for source coding with a fidelity criterion," *IEEE Trans. on Information Theory*, Vol. 29, pp. 197-199, March 1974

I. Csiszár, J. Körner, K. Marton, "A new look at the error exponent of coding for discrete memoryless channels," *IEEE Symposium on Information Theory*, Oct. 1977

- **Isomorphism:**

K. Marton, The problem of isomorphy for general discrete memoryless sources, *Z. Wahrscheinlichkeitstheorie verw. Geb.*, 53. pp. 51-58, 1983

- **Entropy and capacity of graphs:**

J. Körner, K. Marton, "Random access communication and graph entropy," *IEEE Trans. on Inform. Theory*, Vol. 34, No. 2, 312-314, 1988

K. Marton, "On the Shannon capacity of probabilistic graphs," *J. of Combinatorial Theory*, 57, pp. 183-195, 1993

Blowing-Up Lemma

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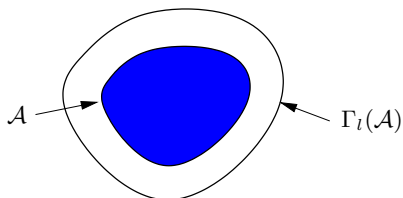
Blowing-Up Lemma

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- Lemma first proved by Ahlswede, Gacs, Körner (1976)
- Used to prove strong converse, e.g., for degraded DM-BC
- Complicated, combinatorial proof

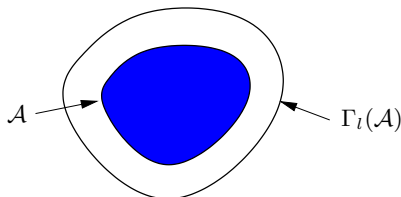
Blowing-Up Lemma

- Let $x^n, y^n \in \mathcal{X}^n$ and $d(x^n, y^n)$ be Hamming distance between them
- Let $\mathcal{A} \subseteq \mathcal{X}^n$. For $l \leq n$, let $\Gamma_l(\mathcal{A}) = \{x^n : \min_{y^n \in \mathcal{A}} d(x^n, y^n) \leq l\}$



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Blowing up Lemma

Let $X^n \sim P_{X^n} = \prod_{i=1}^n P_{X_i}$ and $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$. There exist $\delta_n, \eta_n \rightarrow 0$ as $n \rightarrow \infty$ such that if $P_{X^n}(\mathcal{A}) \geq 2^{-n\epsilon_n}$, then

$$P_{X^n}(\Gamma_{n\delta_n}(\mathcal{A})) \geq 1 - \eta_n$$

Marton's Simple Proof

- The proof uses the following information theoretic coupling inequality

Lemma 1

Let $X^n \sim \prod_{i=1}^n P_{X_i}$ and $\hat{X}^n \sim P_{\hat{X}^n}$. Then, there exists a joint probability measure P_{X^n, \hat{X}^n} with these given marginals such that

$$\frac{1}{n} \mathbb{E}(d(X^n, \hat{X}^n)) = \frac{1}{n} \sum_{i=1}^n \mathbb{P}\{X_i \neq \hat{X}_i\} \leq \left(\frac{1}{n} D \left(P_{\hat{X}^n} \parallel \prod_{i=1}^n P_{X_i} \right) \right)^{1/2}$$

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- Now, define

$$P_{\hat{X}^n}(x^n) = P_{X^n|\mathcal{A}}(x^n) = \begin{cases} \frac{P_{X^n}(x^n)}{P_{X^n}(\mathcal{A})} & \text{if } x^n \in \mathcal{A}, \\ 0 & \text{if } x^n \notin \mathcal{A} \end{cases}$$

Then,

$$D\left(P_{\hat{X}^n} \parallel \prod_{i=1}^n P_{X_i} \right) = -\log P_{X^n}(\mathcal{A}) \leq n\epsilon_n$$

- By Lemma 1, there exists P_{X^n, \hat{X}^n} with given marginals such that

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- By the Markov inequality,

$$P_{X^n, \hat{X}^n} \{d(X^n, \hat{X}^n) \leq n\delta_n\} \geq 1 - \frac{\sqrt{\epsilon_n}}{\delta_n} = 1 - \eta_n,$$

where we choose $\delta_n \rightarrow 0$ such that $\eta_n \rightarrow 0$ as $n \rightarrow \infty$

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- We therefore have

$$P_{X^n}(\Gamma_{n\delta_n}(\mathcal{A})) = P_{X^n, \hat{X}^n}(\Gamma_{n\delta_n}(\mathcal{A}) \times \mathcal{A}) + P_{X^n, \hat{X}^n}(\Gamma_{n\delta_n}(\mathcal{A}) \times \mathcal{A}^c)$$

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- Nair and I in 2008 showed that the extension of their superposition region to > 2 receivers is not optimal in general

Acknowledgments

- Yeow-Khiang Chia
- Lei Zhao